**Appendix A. Description of the Norton Sound Red King Crab Model**

**a. Model description.**

The model is an extension of the length-based model developed by Zheng et al. (1998) for Norton Sound red king crab. The model has 6 male length classes with model parameters estimated by the maximum likelihood method. The model estimates abundances of crabs with CL ≥74 mm and with 10-mm length intervals (6 length classes) because few crabs measuring less than 74 mm CL were caught during surveys or fisheries and there were relatively small sample sizes for trawl and winter pot surveys. The model treats newshell and oldshell male crabs separately but assumes they have the same molting probability and natural mortality.

Calendar Year

Model Year

Winter Fishery

Nov-May

**Jul 1/Y**

Pre-fishery

Abundance

**Jan 1/Y**

**Jan 1/Y-1**

Summer Fishery

Jun -Aug

Molting

Recruitment

CPT Model Assessment

Projection

**May/Y**

CPT Model

Assessment

Projection

**Sept/Y**

**Jan 1/Y+1**

Summer+Winter Fishery

Retained OFL, ABC, GHL

Winter fishery + natural mortality

Summer fishery + natural mortality

natural mortality

**Jul 1/Y+1**

Pre-fishery

Projected

Abundance

MMB

**Feb 1/Y**

Pre-fishery

Projected

Abundance

**Feb 1/Y-1**

Pre-fishery

Abundance

Timeline of calendar events and crab modeling events.

**In this assessment model, model year starts July 1st to June 30th of the following year.**

**Model year starts in July 1st 1976.**

*Initial pre-fishery summer crab abundance on July 1st 1976*

Abundance of the initial pre-fishery population was defined as

|  |  |
| --- | --- |
|  | (1) |

The length proportion of the first year was calculated as

|  |  |
| --- | --- |
|  | (2) |

for model estimated parameters ai. Length-class abundance is the product of first year total abundance length-class proportion. To reduce the number of estimated parameters it is assumed that all crab are newshell in the first year.

*Crab abundance on July 1st*

Summer (01 July) crab abundance of new and oldshells consists of survivors of winter commercial and subsistence crab fisheries and natural mortality from 01Feb to 01July:

|  |  |
| --- | --- |
|  | (4) |

where

*Ns,l,t* , *Os,l,t* : summer abundances of newshell and oldshell crabs in length class *l* in year *t* ,

*Nw,l,t-1*, *Ow,l,t-1* : winter abundances of newshell and oldshell crabs in length class *l* in year *t-1* ,

*Cw,t-1*, *Cp,t-1* : total winter commercial and subsistence catches in year *t-1*,

*Pw,n,l,t-1*, *Pp,n,l,t-1* : Length proportion of winter commercial and subsistence catches for newshell crabs for length class *l* in year *t-1,*

*Pw,o,l,t-1*, *Pp,o,l,t-1* : length compositions of winter commercial and subsistence catches for oldshell crabs in length class *l* in year *t-1,*

*Dw,n,l,t-1*, *Dp,n,l,t-1* : Discard mortality in winter commercial and subsistence catches for newshell crabs in length class *l* in year *t-1,*

*Dw,o,l,t-1*, *Dp,o,l,t-1* : Discard mortality in winter commercial and subsistence catches for oldshell crabs in length class *l* in year *t* -1,

*Ml* : instantaneous natural mortality in length class *l*

0.42 : proportion of the year from Feb 1 to July 1 is 5 months, or 0.42 year.

*Crab abundance on Feb 1st*

Abundance of newshell crabs of year t and length-class l (*Nw,l,t* ) year-t new and oldshell crab that survived the summer commercial fishery and then molted, plus recruitment . Molting is assumed to occur in September, after completion of the fishery, so that

|  |  |
| --- | --- |
|  | (5) |

Length-class bundance of oldshell crabs *Ow,l,t*  consists of the non-molting portion of survivors from the summer fishery:

|  |  |
| --- | --- |
|  | (6) |

where

*Gl’, l* : a growth matrix representing the expected proportion of crabs growing from length class *l’* to length class *l,*given that they have molted;

*Cs,t* : total summer catch in year *t* (assumed to be known without error),

*Ps,n,l,t* , *Ps,o,l,t* : compositions of summer catch for newshell and oldshell crabs in length class *l* in year *t*,

*Dl,t*: summer discard mortality in length class *l* in year *t*,

*ml* : molting probability in length class *l*,

*yc* : the time in year from July 1 to the mid-point of the summer fishery,

0.58: Proportion of the year from July 1st to Feb 1st is 7 months is 0.58 year,

*Rl,t*: recruitment into length class *l* in year *t*.

*Discards*

In summer and winter commercial fisheries, sublegal males (<4.75 inch CW and <5.0 inch CW since 2005) are discarded. Those discarded crabs are subject to handling mortality.

Model discard mortality in length-class *l* in year *t* from the summer and winter commercial pot fisheries is given by

|  |  |
| --- | --- |
|  | (7) |
|  | (8) |
|  | (9) |
|  | (10) |
|  | (11) |

where

*hms*: summer commercial handling mortality rate assumed to be 0.2,

*hmw*: winter commercial handling mortality rate assumed to be 0.2,

*Ll* : the proportion of legal males in length class *l*,

*Ss,l*: Selectivity of the summer commercial fishery,

*Cd,t*: Winter subsistence discards catch,

*Pd,n,l,t* , *Pd,o,l,t* : Compositions of discards for newshell and oldshell crabs in length class *l* in year *t*.

*Recruitment*

Recruitment of year *t*, *Rt*, is a stochastic process around the geometric mean, *R0*:

|  |  |
| --- | --- |
|  | (14) |

*Rt* of the last year was assumed to be an average of previous 5 years: *Rt* = (*Rt-1* + *Rt-2* + *Rt-3* + *Rt-4* + *Rt-5* )/5.

*Rt* was assumed to come from only length classes 1 and 2 so that

|  |  |
| --- | --- |
|  | (15) |

where *r* is a positive parameter with a value less than or equal to 1. *Rl,t* = 0 when *l* ≥ 3.

*Molting Probability*

Molting probability for length class *l*, *ml*, was fitted as a decreasing logistic function of length-class mid carapace length and constrained to equal 0.99 for the smallest length-class (*L1*):

|  |  |
| --- | --- |
|  | (12) |

*Trawl net and pot selectivity*

For efficiency of estimating model parameters, the above equation was modified, so that selectivity reaches 0.99 at the mid-length of the largest lengths class (*L6*)

|  |  |
| --- | --- |
|  | (13’) |

For summer trawl survey, two selectivity curves with parameters (*φt1*, *φt2*) were estimated: 1) during NMFS survey 1976-1991, and 2) during ADF&G survey since 1996. Similarly, two selectivity curves with parameters (*φ1*, *φ2*) were estimated for the summer commercial fishery: 1) before 1993, and 2) 1933 to present reflecting changes in fisheries, and crab pot configurations.

For winter pot survey and winter harvest parameter (*φw*), selectivity (*Sw,l*) was assumed to be dome shaped, with *Sw,5* =0.999, and *Sw,6*  was directly estimated from the model.

*Growth transition matrix*

The growth matrix *Gl’, l* (the expected proportion of crabs molting from length class *l’* to length class *l* ) was

Growth matrix was assumed to be normally distributed

|  |  |
| --- | --- |
|  | (14) |

Where



For approximation of , we applied Simpson’s approximate integration rule 

Observation model

Estimates of survey abundances

*Summer trawl survey abundance*

Modeled trawl survey abundance of *t-th* year (*Bst,t*) is July 1st abundance subtracted by summer commercial fishery harvest occurring from the July 1st to the mid-point of summer trawl survey, multiplied by natural mortality occurring between mid-point of commercial fishery date and trawl survey date, and multiplied by trawl survey selectivity. For the first year (1976) trawl survey, commercial fishery did not occur.

|  |  |
| --- | --- |
|  | (15) |

Where

*yst* : the time in year from July 1 to the mid-point of the summer trawl survey,

(*yst* > *yc*: Trawl survey starts after opening of commercial fisheries),

*Pc,t* : proportion of summer commercial crab harvested before the mid-point of trawl survey date.

*Winter pot survey CPUE*

Winter pot survey cpue (*fwt*) was calculated with catchability coefficient *q* and exploitable abundance

|  |  |
| --- | --- |
|  | (16) |

*Summer pot survey abundance* (Removed from likelihood components)

Abundance of *t-*th year pot survey was estimated as

|  |  |
| --- | --- |
|  | (17) |

Where

*yp* : the time in year from July 1 to the mid-point of the summer trawl survey.

Summer commercial CPUE

Summer commercial fishing CPUE (*ft*) was calculated as a product of catchability coefficient *q* and mean exploitable abundance minus one half of summer catch,At.

|  |  |
| --- | --- |
|  | (18) |

Because fishing fleet and pot limit configuration changed in 1993 *q1* is for fishing efforts before 1993, *q2* is from 1994 to present.

Where *At* is exploitable legal abundance in year *t*, estimated as

|  |  |
| --- | --- |
|  | (24) |

Estimates of length composition

*Summer commercial catch*

Length compositions of the summer commercial catch for new and old shell crabs *Ps,n,l,t*and *Ps,o,l,t*, were modeled based on summer population, selectivity, and legal abundance;

|  |  |
| --- | --- |
|  | (23) |

*Summer commercial fishery Observer discarddata*

Length/shell compositions of observer discards in 87-90, 92, 94, and 2012 were modeled as

|  |  |
| --- | --- |
|  | (25) |

*Summer trawl survey*

Some trawl surveys occurred during the molting period, and thus we combined the length compositions of newshell and oldshell crabs as one single shell condition, *Pst,l,t*, given by

|  |  |
| --- | --- |
| = | (26) |

*Summer pre-season survey (1976)* (Removed from likelihood due to only 1 year of survey)

The same selectivity for the summer commercial fishery was applied to the summer pre-season survey, resulting in estimated length compositions for both newshell and oldshell crabs as:

|  |  |
| --- | --- |
|  | (27) |

This was not incorporated into likelihood calculation because of one year data.

*Summer pot survey (1980-82, 85)* (Removed from likelihood with failure to locate original data)

The length/shell condition compositions of summer pot survey were estimated as

|  |  |
| --- | --- |
|  | (28) |

*Winter commercial catch*

Length compositions of winter commercial catch (*Pw,n,l,t*, *Pw,o,l,t*) for length *l* in year *t* were estimated from the winter population, winter pot selectivity, and proportion of legal crabs for each length class as:

|  |  |
| --- | --- |
|  | (19) |

*Winter subsistence catch*

Subsistence fishery does not have a size limit; however, crabs of size smaller than length class 3 are generally not retained. Hence, we assumed proportion of length composition *l* = 1 and 2 as 0, and estimated length compositions (*l* ≥ 3) as follows

|  |  |
| --- | --- |
|  | (20) |

*Winter subsistence discards*

Subsistence fishery discards proportion was assumed to be length composition *l* = 1 and 2 only, and was estimated as follows

|  |  |
| --- | --- |
|  | (21) |

*Winter pot survey*

Winter pot survey length compositions for newshell and oldshell crabs, *Psw,n,l,t*and *Psw,o,l,t* (*l* ≥ 1) were calculated as

|  |  |
| --- | --- |
|  | (22) |

*Estimates of tag recovery*

The proportion of tagged length class *l’* crabs released was recovered after *t-th* year with length class of *l* by a fishery of *s-th* selectivity (Sl) was assumed proportional to growth matrix, catch selectivity, and molting probability (*ml*) as

|  |  |
| --- | --- |
|  | (29) |

where **X** is a molting probability adjusted growth matrix with each component consists of

|  |  |
| --- | --- |
|  | (30) |

**b. Software used**: AD Model Builder (Fournier et al. 2012).

**c. Likelihood components.**

Under assumptions that measurement errors of annual total survey abundances and summer commercial fishing efforts follow lognormal distributions and each type of length composition has a multinomial error structure (Fournier and Archibald 1982; Methot 1989), the log-likelihood function is:

|  |  |
| --- | --- |
|  | (31) |

1where

*i*: length/shell compositions of :

1 triennial summer trawl survey,

2 annual winter pot survey,

3 summer commercial fishery,

4 observer discards during the summer fishery.

*ni*: the number of years in which data set *i* is available,

*Ki,t*: the effective sample size of length/shell compositions for data set *i* in year *t,*

*Pi,l,t* : observed and estimated length compositions for data set *i*, length class *l*, and year *t.*

In this, while observation and estimation were made for oldshell and newshell separately, both were combined for likelihood calculations.

*κ* : a constant equal to 0.001,

*CV* : coefficient of variation for the survey abundance,

*Bi,k,t*: observed and estimated annual total abundances for data set *i* and year *t,*

*ft* : observed and estimated summer fishing CPUE,

*w2t*: extra variance factor,

*fwt* : observed and estimated winter survey CPUE,

*SDRw* : Standard deviation of winter survey CPUE = 0.3,

*SDR* : Standard deviation of recruitment = 0.5,

*Kl’,t*: the effective sample size of length class *l’* released and recovered after *t-th* in year,

*Kl’,t*: the effective sample size of length class *l’* released and recovered after *t-th* in year,

*Pl’,l,t,s* : observed and estimated proportion of tagged crab released at length *l’* and recaptured at

length *l*, after *t-th* year by commercial fishy pot selectivity *s*,

*s*: fishery selectivity (1) 1976-1992, (2) 1993- present,

It is generally believed that total annual commercial crab catches in Alaska are fairly accurately reported. Thus, total annual catch was assumed known.

**e. Parameter estimation framework:**

1. Parameters Estimated Independently

The following parameters were estimated independently: natural mortality (*M* =0.18), proportions of legal males by length group, and the growth transition matrix (model 0).

Natural mortality was based on an assumed maximum age, *tmax*, and the 1% rule (Zheng 2005):

,

where *p* is the proportion of animals that reach the maximum age and is assumed to be 0.01 for the 1% rule (Shepherd and Breen 1992, Clarke et al. 2003). The maximum age of 25, which was used to estimate *M* for U.S. federal overfishing limits for red king crab stocks (NPFMC 2007) results in an estimated *M* of 0.18. Among the 199 recovered crabs from the tagging returns during 1991-2007 in Norton Sound, the longest time at liberty was 6 years and 4 months from a crab tagged at 85 mm CL. The crab was below the mature size and was likely less than 6 years old when tagged. Therefore, the maximum age from tagging data is about 12, which does not support the maximum age of 25 chosen by the CPT.

Proportions of legal males (CW > 4.75 inches) by length group were estimated from the ADF&G trawl data 1996-2011 (Table 8).

Mean growth increment per molt, standard deviation for each pre-molt length class, and the growth matrix (Table 8), were estimated from tagging surveys conducted in summer 1981-1985, and winter 1981-present. In summer 1981-1985 study legal and sublegal males captured by the survey pots were tagged, and in the1981-present winter survey, sublegal males were tagged. All tagged crabs were recaptured by summer and winter commercial/subsistence fisheries.

1. Parameters Estimated Conditionally

Estimated parameters are listed in Table 10. Selectivity and molting probabilities based on these estimated parameters are summarized in Tables 12 and 13.

A likelihood approach was used to estimate parameters (Table 11)

**f. Definition of model outputs.**

1. Mature Male Biomass (MMB) on July 1st was defined as consisting of the biomass of male crabs in length classes of 3 to 6



*wml*: mean weight of each length class (Table 12).

1. Projected Legal Male Biomass for winter and summer fishery OFL was calculated as



1. Recruitment: the number of males of the length classes 1 and 2.